Phase diagram of electron system in vicinity of superconductor-insulator transition

Thomas Nattermann
University of Cologne

in collaboration with Valery Pokrovsky and Gianmaria Falco

Support from the grants DFG NA222/5-2, SFB 680 D4 and DOE DE-FG02-06ER46278

arXiv:1001.5431v1 [cond-mat.supr-con] 29 Jan 2010

PRL **100**, 060402 (2008) PR B 80, 104515 (2009)
EPL **85**, 30002 (2009)

analytical approach

well separated length and energy scales

weak disorder, unbounded

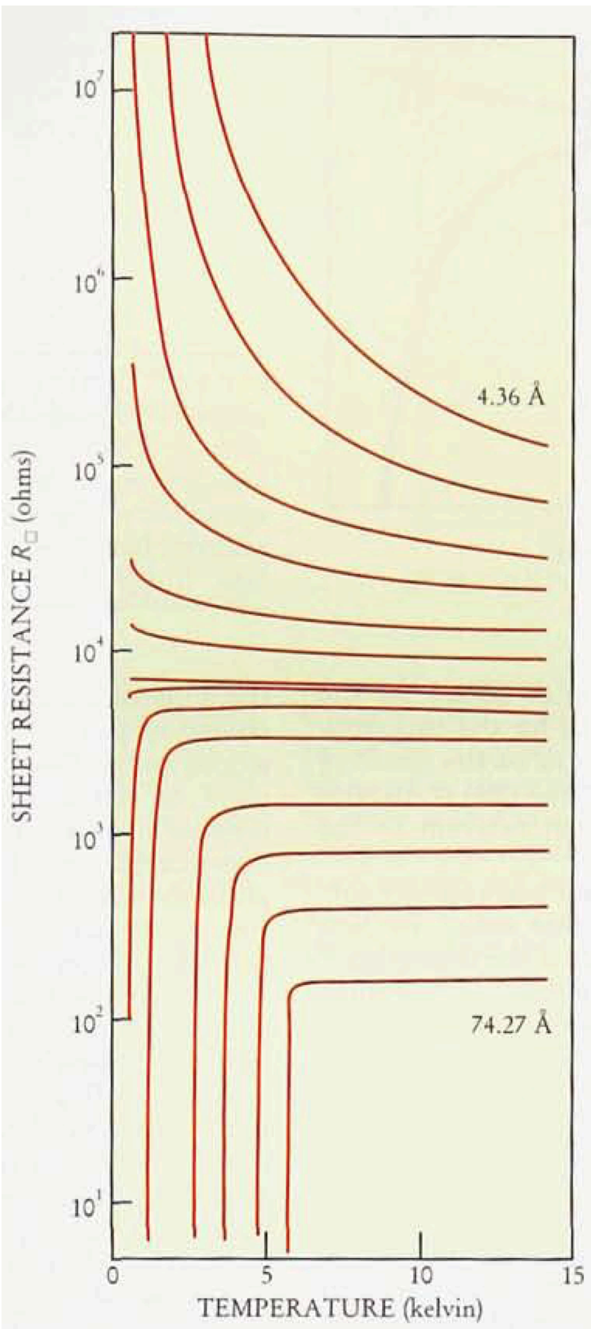
zero temperature

Outline

- Experimental facts
- Theoretical works
- Model and method
- Thin film in parallel magnetic field
- Thin film in perpendicular magnetic field
- Thick film
- Resistance

2D SC/I quantum phase transition

Sheet Resistance (W)



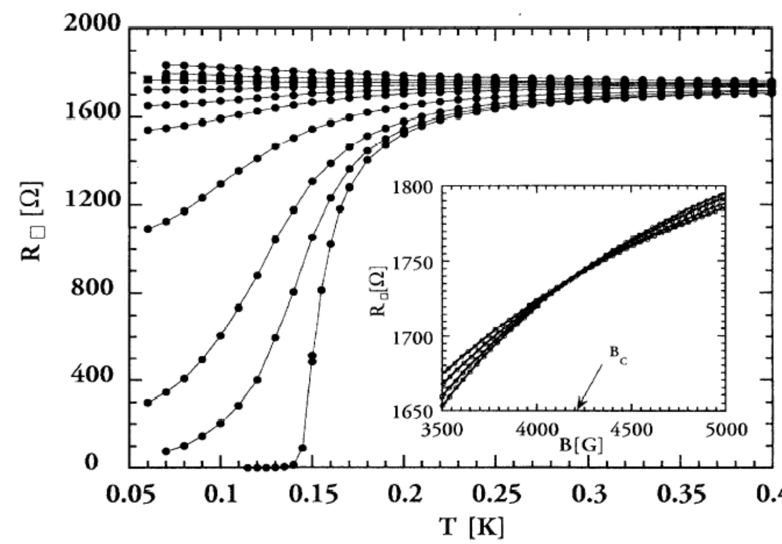


FIG. 1. Zero bias resistance of sample 2 plotted versus temperature at $B = 0, 0.5, 1.0, 2.0, 3.0, 4.0, 4.4, 4.5, 5.5, 6$ kG. In the inset, $R_\square(B,T,E=0)$ for the same sample measured versus field, at $T = 80, 90, 100, 110$ mK.

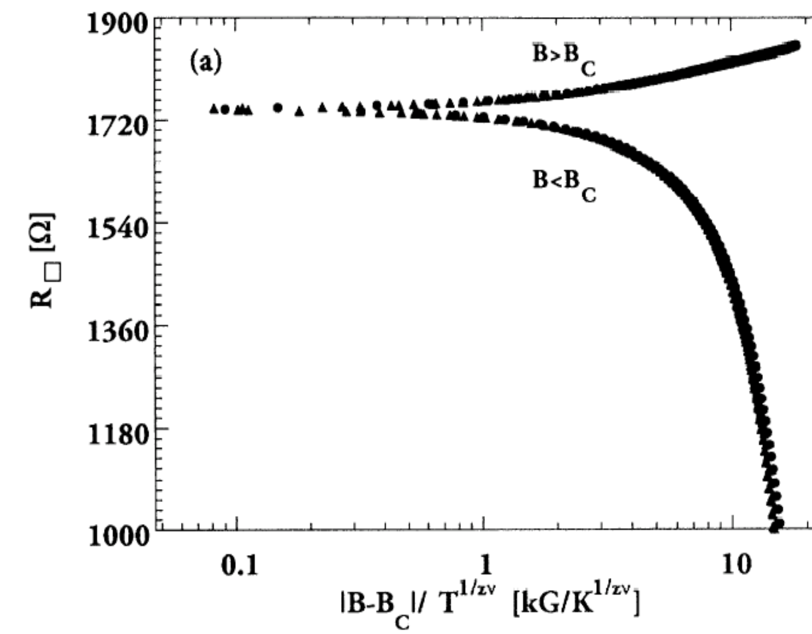


FIG. 3. Top: Scaling of $R_\square(B,T,E=0)$ for sample 2 measured at $T = 80, 90, 100, 110$ mK ($B_c = 4.19$ kG, $\nu z = 1.36$).

Yazdani and Kapitulnik, **Phys. Rev. Lett. 74, 3037–3040 (1995)**

Two-Dimensional a-MoGe Thin Films

Goldman and Markovic, Physics Today 1998, amorphous very thin Bi films (near 10)

Localized Superconductivity in the Quantum-Critical Region of the Disorder-Driven Superconductor-Insulator Transition in TiN Thin Films

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(Received 7 May 2007; published 18 December 2007)

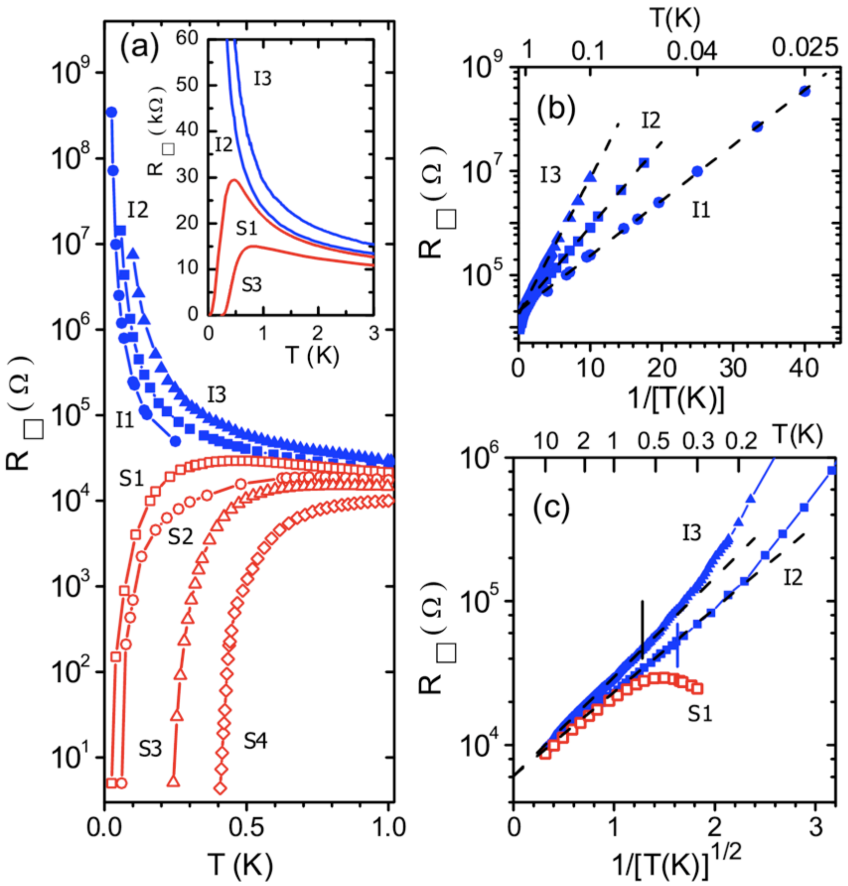

FIG. 1 (color online). Temperature dependences of $R_\square$ taken at zero magnetic field for the samples near the localization threshold. (a) $\log R_\square$ versus T. Inset: some part of the $R_\square$ data in a linear scale. (b) $\log R_\square$ versus $1/T$ for samples I1, I2, and I3. Dashed lines represent Eq. (1) and fit perfectly at low temperatures. All curves saturate at the same $R_\square \approx 20$ kΩ at high temperatures. (c) $R_\square$ versus $1/T^{1/2}$; dashed lines are given by $R_\square = R_1 \exp(T_1/T)^{1/2}$ which (with $R_1 \sim 6$ kΩ) well fit the data at high temperatures. Vertical strokes mark T_0, determined by the fit to the Arrhenius formula of Eq. (1).

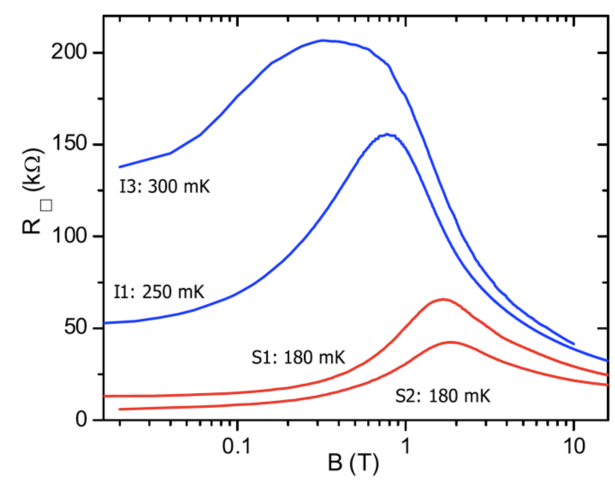

FIG. 2 (color online). Magnetoresistance isotherms for conducting (S1, S2) and insulating samples (I1, I3) at temperatures. All curves converge above 2 T.

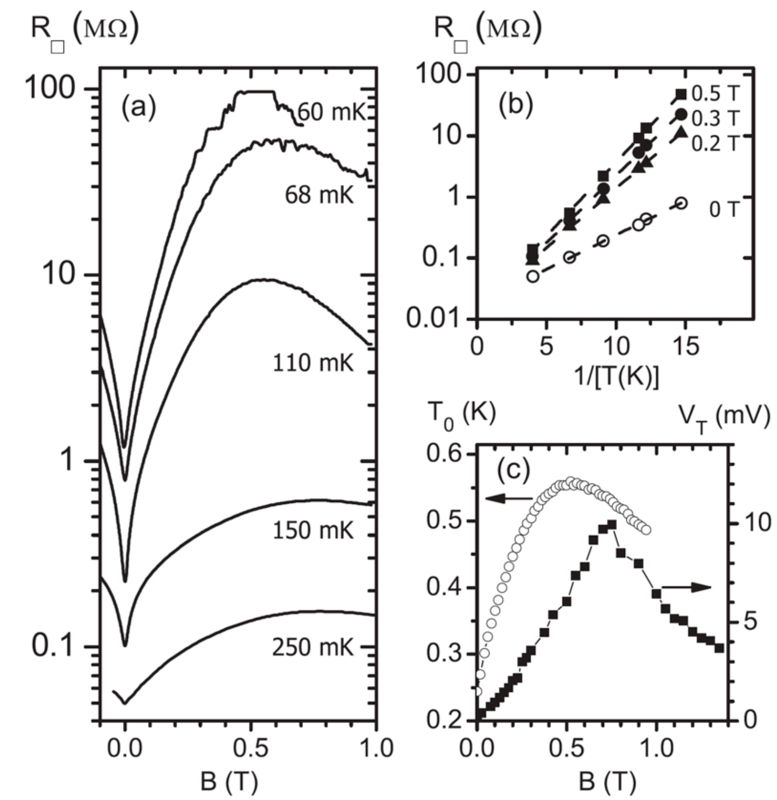

FIG. 3. (a) Sheet resistance of sample I1 as a function of the magnetic field at some temperatures listed. (b) R versus $1/T$ at $B = 0$ (open circles), 0.2 (triangles), 0.3 (filled circles), and 0.5 T (squares). The dashed lines are given by Eq. (1). (c) T_0 (left axis), calculated from fits to Eq. (1), and the threshold voltage V_T (right axis) as a function of B.

Giant negative magnetoresistance (GNM)

VOLUME 85, NUMBER 1 PHYSICAL REVIEW LETTERS 3 JULY 2000

Tenfold Magnetoconductance in a Nonmagnetic Metal Film

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Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70806

(Received 9 November 1999)

We present magnetoconductance (MC) measurements of homogeneously disordered Be films whose zero field sheet conductance (G) is described by the Efros-Shklovskii hopping law $G(T) = (2e^2/h)\exp-(T_0/T)^{1/2}$. The low field MC of the films is negative with G decreasing a factor of 2 below 1 T. In contrast the MC above 1 T is strongly positive. At 8 T, G increases tenfold in perpendicular field and fivefold in parallel field. In the simpler parallel case, we observe *field enhanced* variable range hopping characterized by an attenuation of T_0 via the Zeeman interaction.

PACS numbers: 72.20.Ee, 71.30.+h, 73.50.−h

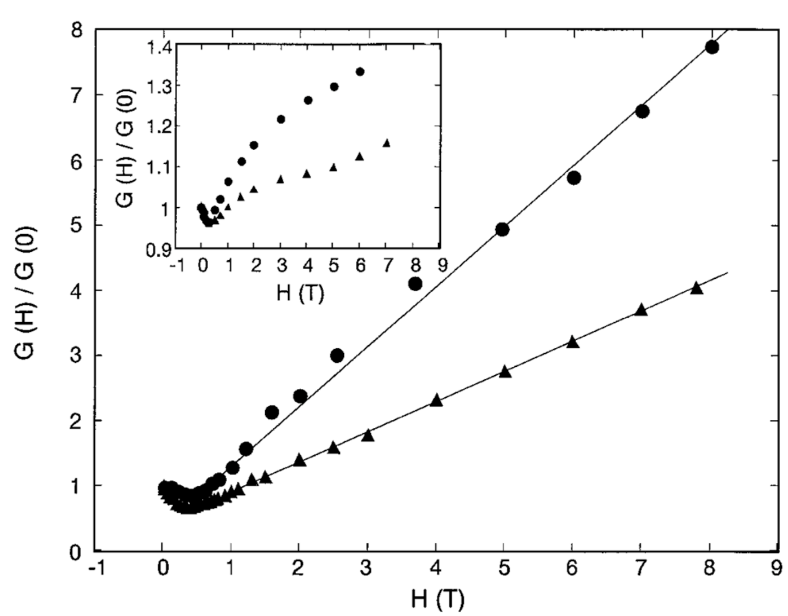
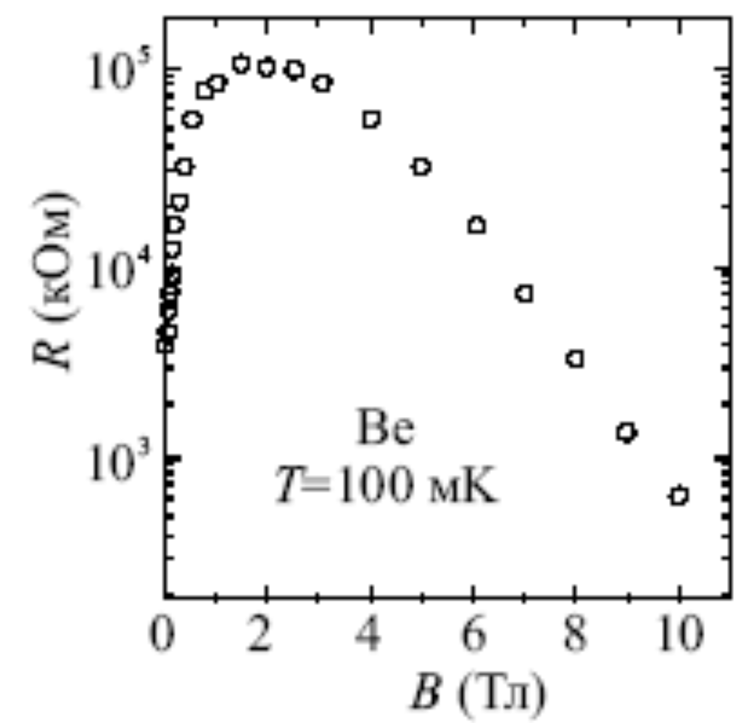


FIG. 1. Relative magnetoconductance of a 3 MΩ Be film at 50 mK. Circles: field perpendicular to film surface. Triangles: field parallel to film surface. The solid lines are linear fits to the data above 1 T with slopes of 1/(1.1 T) and 1/(2.2 T) for the perpendicular and parallel data, respectively. Inset: relative magnetoconductance of a 16 kΩ Be film.



Wenhao Wu, AIP Conference Proceeding (LT24) **850**, 995 (2006)

Be thin films

Amorphous films In_2O_{3-x}

JETP Letters, Vol. 71, No. 4, 2000, pp. 160–164. From Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 71, No. 4, 2000, pp. 231–237.

CONDENSED MATTER

Scaling Analysis of the Magnetic Field–Tuned Quantum Transition in Superconducting Amorphous In–O Films[1]

V. F. Gantmakher*, M. V. Golubkov, V. T. Dolgopolov,
G. E. Tsydynzhapov, and A. A. Shashkin
Institute of Solid-State Physics, Russian Academy of Sciences, Chernogolovka, Moscow oblast, 142432 Russia
**e-mail: gantm@issp.ac.ru*
Received January 27, 2000

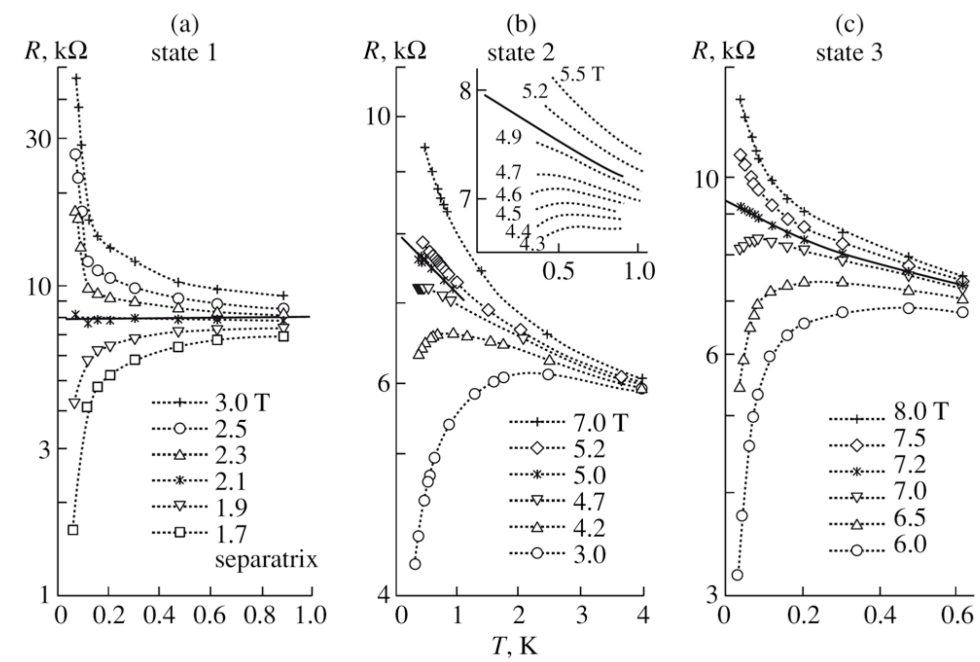


Fig. 1. Temperature-dependent resistances for the states studied at different magnetic fields. The separatrices $R_c(T)$ are shown by solid lines. For state 2, a close-up view of the critical region is displayed in the inset.

JETP Letters, Vol. 71, No. 11, 2000, pp. 473–476. From Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 71, No. 11, 2000, pp. 693–697.

Observation of the Parallel-Magnetic-Field-Induced Superconductor–Insulator Transition in Thin Amorphous InO Films[1]

V. F. Gantmakher, M. V. Golubkov, V. T. Dolgopolov, A. A. Shashkin, and G. E. Tsydynzhapov
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Received April 25, 2000

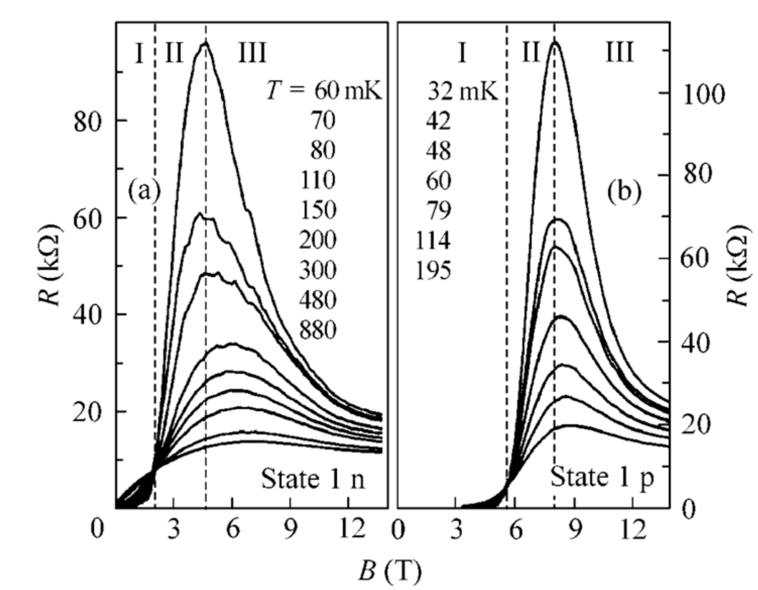

Fig. 1. Isotherms $R(B)$ for (a) normal and (b) parallel magnetic field. The dashed lines separate regions I, II, and III and to the critical field B_c and the resistance maximum at the lowest temperatures.

Temperature dependence of resistance in two samples at different magnetic fields. Extracted from the works:

Magnetic field dependence of resistance in one sample at different starting deviations from the SIT on the insulating side.

Temperature behavior of Conductivity: activation at small fields *B*<10T, Mott VRH behavior at larger *B*: $R \sim e^{(T_0/T)^{1/4}}$

Bi amorphous films

PRL **103,** 157001 (2009) PHYSICAL REVIEW LETTERS week ending 9 OCTOBER 2009

Observation of Giant Positive Magnetoresistance in a Cooper Pair Insulator

H. Q. Nguyen,[1] S. M. Hollen,[1] M. D. Stewart, Jr.,[1] J. Shainline,[1] Aijun Yin,[2] J. M. Xu,[2] and J. M. Valles, Jr.[1]
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(Received 23 July 2009; revised manuscript received 18 September 2009; published 5 October 2009)

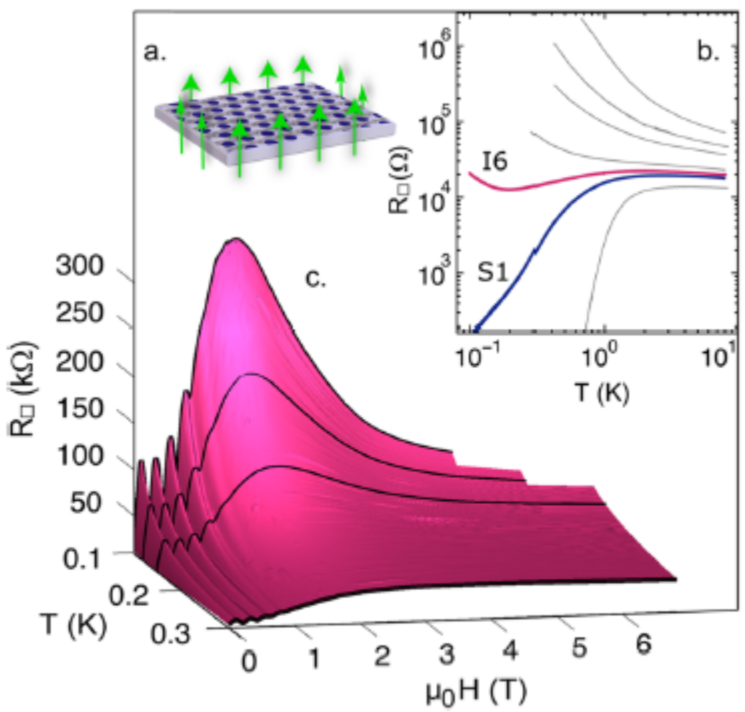


FIG. 1 (color online). (a) SEM image of the nanohoneycomb substrate. The hole center to center spacing and radii are 100 ± 5 and 27 ± 3 nm, respectively. Arrows denote $\vec{H}$. (b) Sheet resistance as a function of temperature, $R_\square(T)$, of NHC films produced through a series of Bi evaporations. The film I6 is the last insulating film and S1 is the first superconducting film in the series. (c) Surface plot of $R_\square(T, H)$ for film I6, which has a normal state sheet resistance of 19.6 kΩ and 1.1 nm Bi thickness. The solid lines are isotherms.

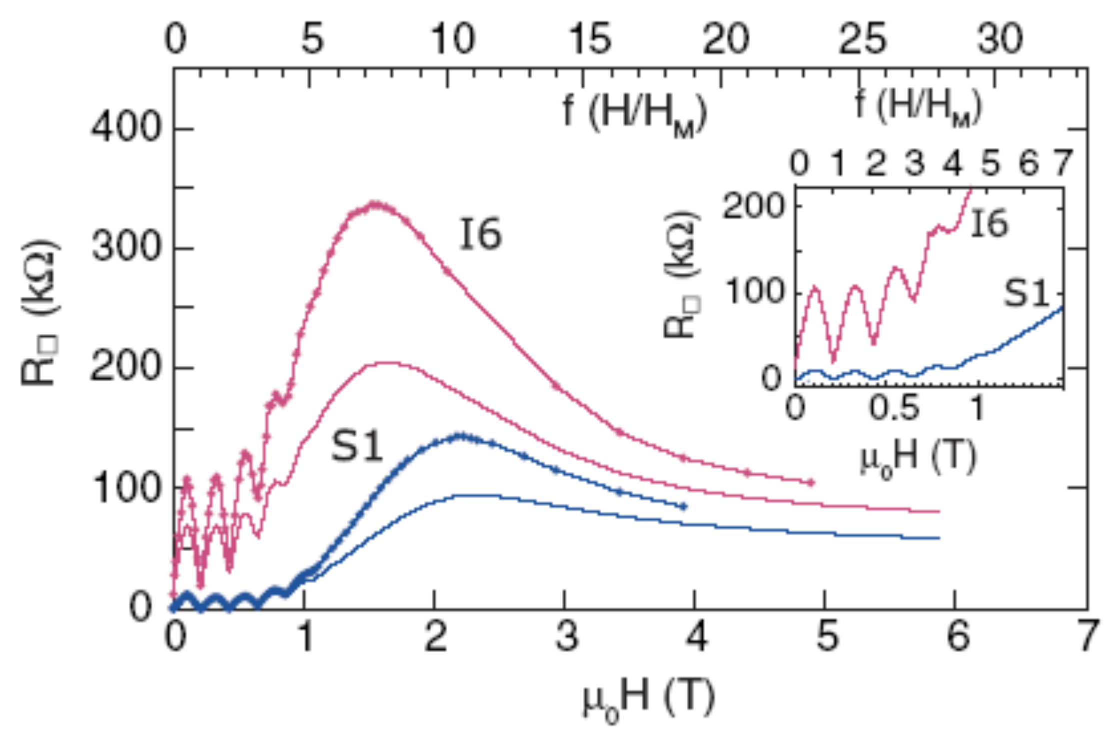
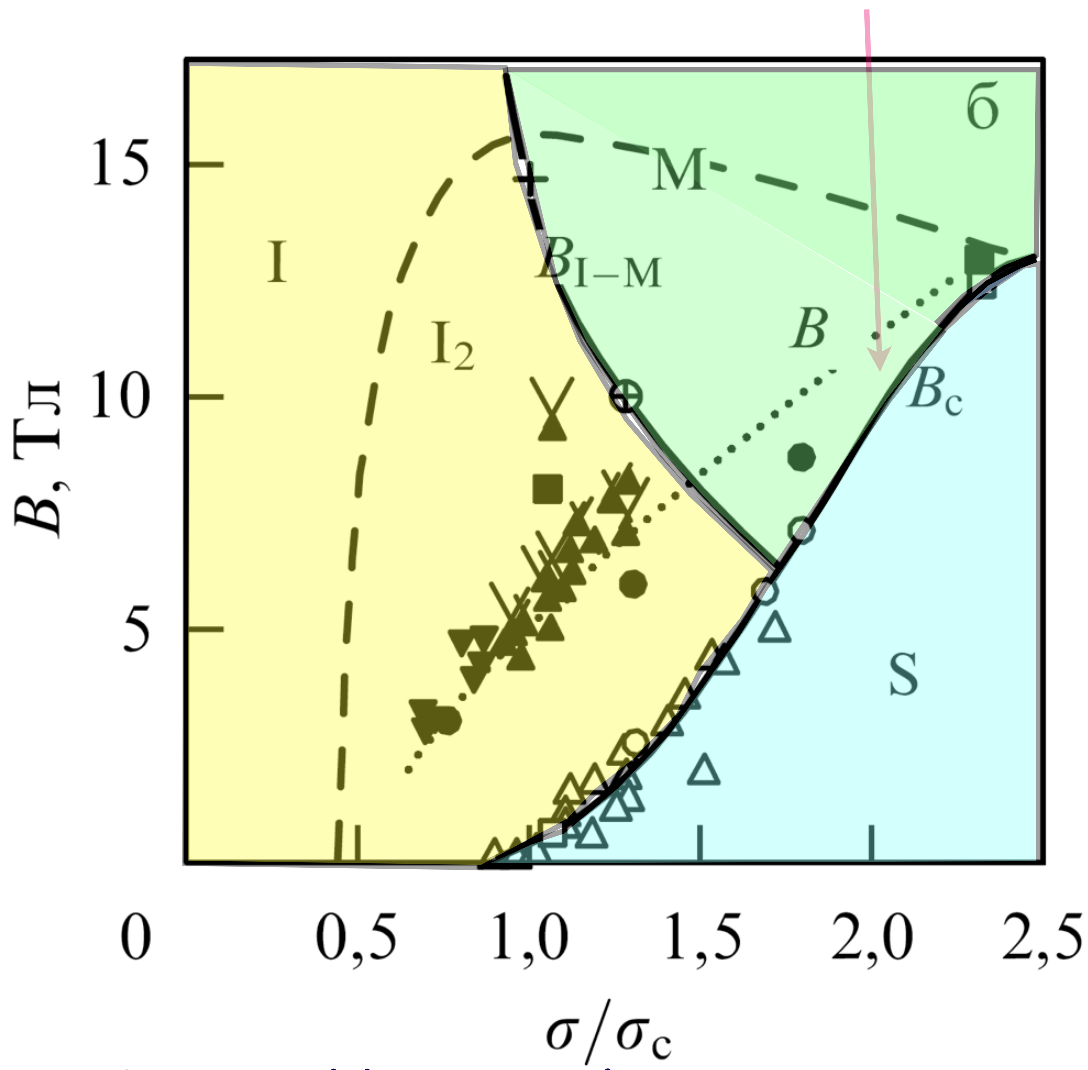
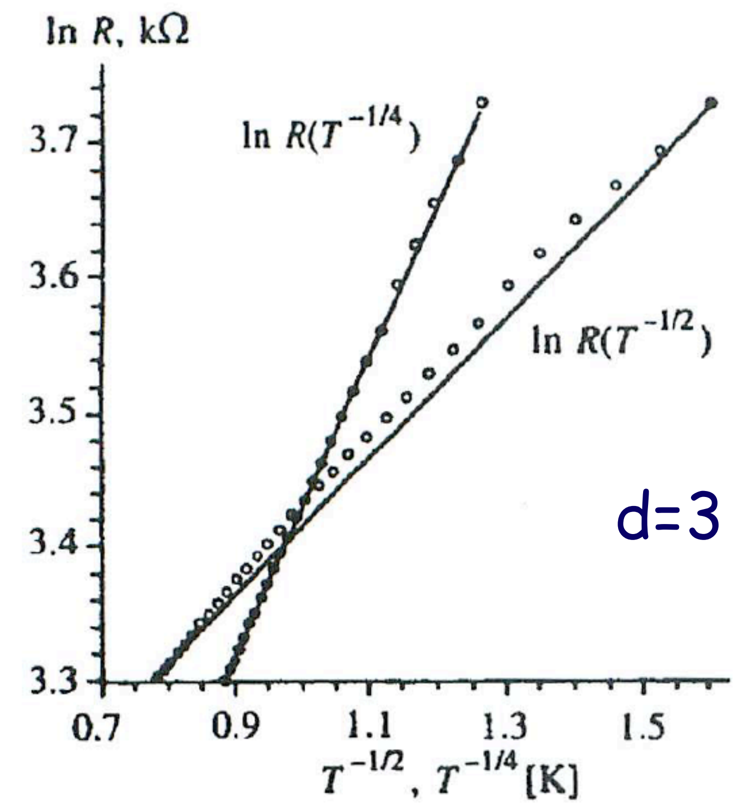


FIG. 2 (color online). Sheet resistance as a function of H at 100 and 120 mK for films I6 and S1. The lines are spline fits to the data points (shown as symbols on the 100 mK traces). Inset: Magnified view of the low H data.

Superconductor to insulator transition of InO, TiN, Bi, Be, high-Tc materials



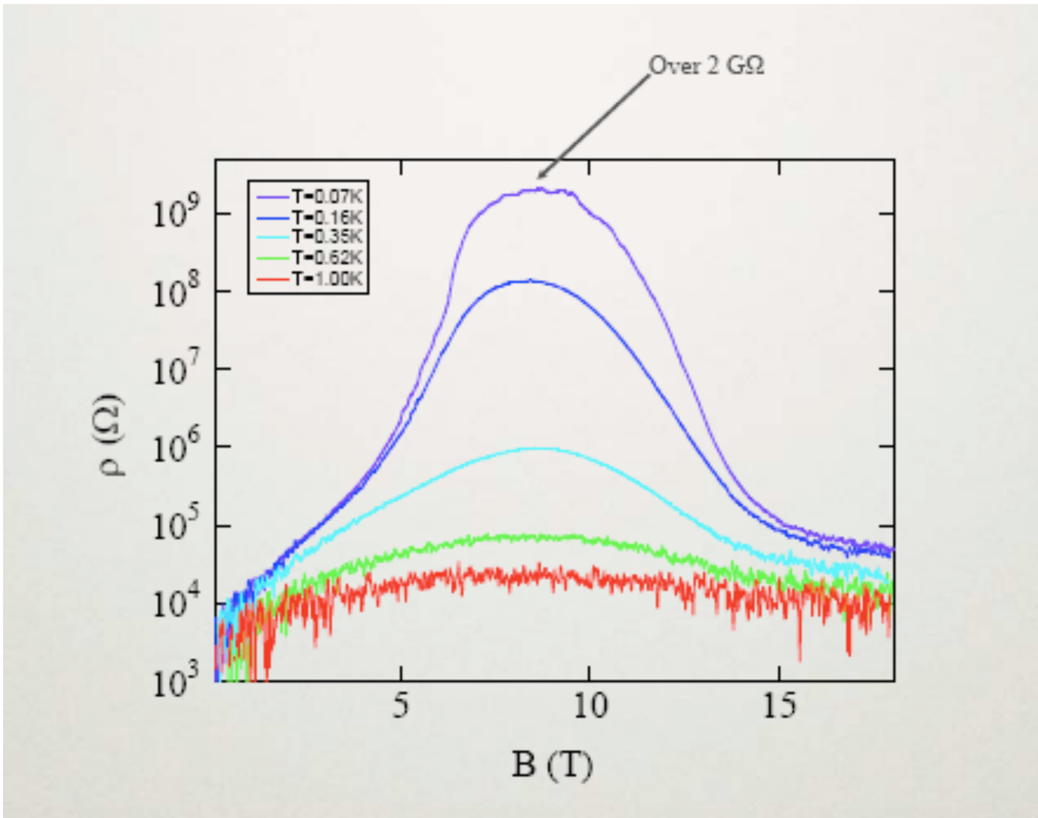


d=3 VRH

Gantmakher et al. 2010

Common believe:

Cooper pairs survive in insulating phase!



Giant negative magnetic resistance

Theoretical works

A. M. Finkelstein, JETP Letters **45**, 46 (1987).

2d , no magnetic field. Coulomb interaction + disorder suppresses superconductivity.
No reasons for GNM.

K. B. Efetov, JETP **78**, 1015 (1981).

Cooper pairs are bound in granules and can tunnel between them. Depending on relative strength of interaction and hopping amplitude the S or I phase is realized.
No real grains in films, the interaction and hopping are not independent

M.P.A. Fisher, Phys. Rev. Lett. 65, 923 (1990).

2d. Duality between vortices and CP. In superconductors CP are free, vortices are bound. In insulators CP are bound, vortices are free. Universal resistance at SIT transition.
Experiments do not confirm the duality and universal resistance.

nature Vol 449|18 October 2007|doi:10.1038/nature06180

LETTERS

Nature of the superconductor–insulator transition in disordered superconductors

Yonatan Dubi[1], Yigal Meir[1,2] & Yshai Avishai[1,2]

PHYSICAL REVIEW B **78**, 024502 (2008)

Island formation in disordered superconducting thin films at finite magnetic fields

Yonatan Dubi,[1,*] Yigal Meir,[1,2] and Yshai Avishai[1,2,3]
[1]*Physics Department, Ben-Gurion University, Beer Sheva 84105, Israel*
[2]*The Ilse Katz Center for Meso- and Nano-scale Science and Technology, Ben-Gurion University, Beer Sheva 84105, Israel*
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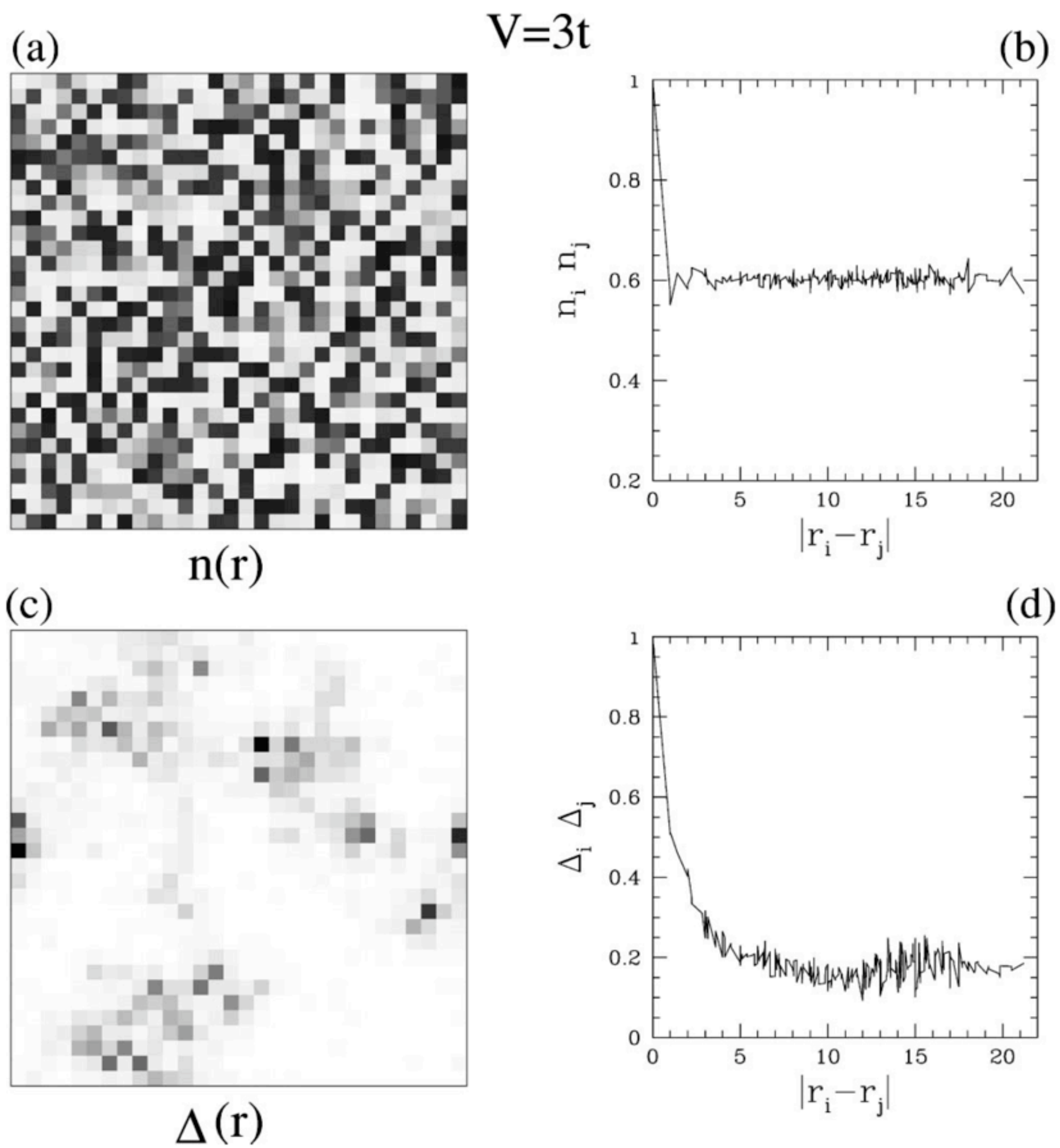
(Received 28 December 2007; published 1 July 2008)

Disappearance of superconducting islands in large magnetic field.

PHYSICAL REVIEW B, VOLUME 65, 014501

Inhomogeneous pairing in highly disordered *s*-wave superconductors

Amit Ghosal, Mohit Randeria, and Nandini Trivedi
Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400005, India
(Received 13 March 2001; published 29 November 2001)



Numerical solution of Bogoliubov-de Gennes equations on a 2d lattice with random distribution of single particle levels. Islands of Cooper pairs.
Comparatively homogeneous electron density. Localization.

Annals of Physics 325 (2010) 1390–1478

Contents lists available at ScienceDirect

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journal homepage: www.elsevier.com/locate/aop

Fractal superconductivity near localization threshold

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[b] Moscow Institute of Physics and Technology, Moscow 141700, Russia
[c] Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854, USA
[d] CNRS and Université Paris-Sud, UMR 8626, LPTMS, Orsay Cedex, F-91405 France
[e] Abdus Salam International Center for Theoretical Physics, Trieste, Italy
[f] Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain

Cooper pairs in insulating phase. Enhanced gap.
Important role of the fractal states near localization threshold.

Purpose of this work:

Explanation of the anomalous magnetic behavior

Construction of complete phase diagram

We show that

There exist 3 different non-superconducting phases:
Bosonic insulator, Fermionic insulator and metal

Transitions between these phases are due either to Zeeman depairing or to squeezing of Cooper pairs by potential wells of disordered potential.

Model assumptions

- Cooper pairs survive in insulator phase. CP have a fixed binding energy Δ
- Near SIT $k_F l \approx 1, \quad l \approx 1nm, \quad n_3 \approx 10^{21} cm^{-3}$
- Fluctuations of electron density on the distance ξ are small (Ghosal et al)
- Number of CP is not conserved, but their average density is well defined $n_b \sim \frac{\Delta}{E_F} n_e$
- CP density *n* is 3 to 4 orders of magnitude smaller than the electron density
- A weak long-scale random potential can localize them and form SC droplets
- Random potential acting on CP is uncorrelated Gaussian
- Coulomb forces on a distance > $n_e^{-1/d}$ are weak due to screening.
- CP can be destroyed either by paramagnetic effect (Zeeman energy exceeds binding energy) or due to squeezing (the size of the droplet becomes of the order of the CP size).

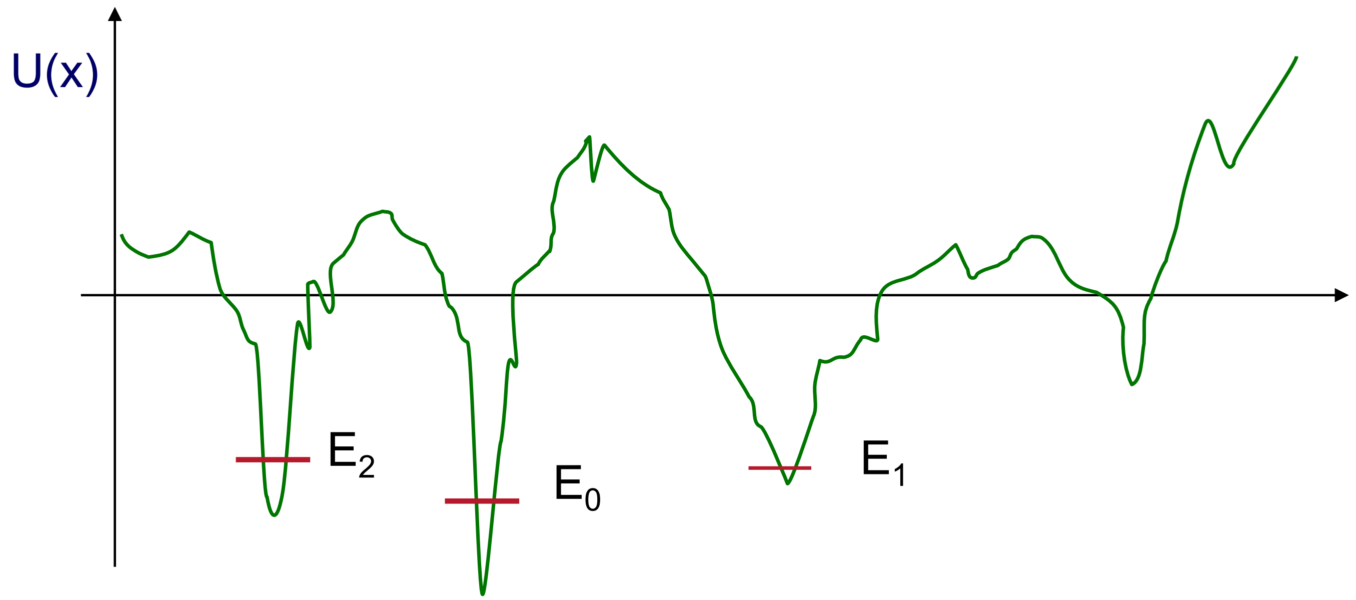
Hamiltonian for bosons / fermions

$$\hat{\mathcal{H}}_k = \frac{1}{2m_k}\left(\mathbf{p} - \frac{e_k}{c}\mathbf{A}\right)^2 + U_k(\mathbf{r}) - g_e\mu_B\mathbf{s}_k\mathbf{B}, \quad k = f, b$$

$$\frac{e_b}{e_f} = \frac{m_b}{m_f} = \frac{U_b}{U_f} = 2 \qquad s_b = 0, \quad s_f = 1/2$$

Random potentials are due to stray electric fields

$$\langle U_k(\mathbf{r})U_k(\mathbf{r}')\rangle = \kappa_k^2\delta(\mathbf{r}-\mathbf{r}')$$



Random potentials are due to stray electric fields

$$\langle U_k(\mathbf{r}) U_k(\mathbf{r}') \rangle = \kappa_k^2 \delta(\mathbf{r} - \mathbf{r}')$$

Larkin length $\mathcal{L}_k = (\hbar^2/(m_k \kappa_k))^{2/(4-d)}$

mean free path or extension of localized states

magnetic length $\ell_k = \left(\dfrac{\hbar c}{e_k B} \right)^{1/2}$

correlation length $\xi = 0.85\sqrt{3\xi_0 l/d}$

$$\mathcal{L}_b \gg \xi \gg l \qquad \mathcal{E}_b = \frac{\hbar^2}{2m_b \mathcal{L}_b^2} \ll \Delta \ll E_F$$

Idea for GNM:

CP pairs fill localized states of the random potential forming Bose-insulator. High magnetic field causes depairing. The appearing fermions are weakly localized.

But how to treat random potential ?

Replica trick $\longrightarrow$ translationally invariant system

Here: method of optimal fluctuation

Density of states, search for the optimal fluctuation of random potential

$$\nu(E) = \int DU \; Tr\, \delta(E - \hat{H}) e^{-\int d^3 r U^2 / 2\kappa^2}$$

$$\int DU \, \exp\left[- \int d^3 r U^2 / 2\kappa^2 + \lambda(E - min_\Psi \langle \Psi | H | \Psi \rangle) \right]$$

$$\boxed{\begin{array}{l} \rightarrow U(\mathbf{r}) = -\lambda \kappa^2 |\Psi|^2 \\ \rightarrow \hat{H}\Psi = E\Psi \qquad \rightarrow \Psi \end{array}} \longrightarrow$$

non-linear Schroedinger equation

I.M. Lifshitz '66,
Zittartz and Langer '66,
Halperin and Lax,'66
Cardy '78

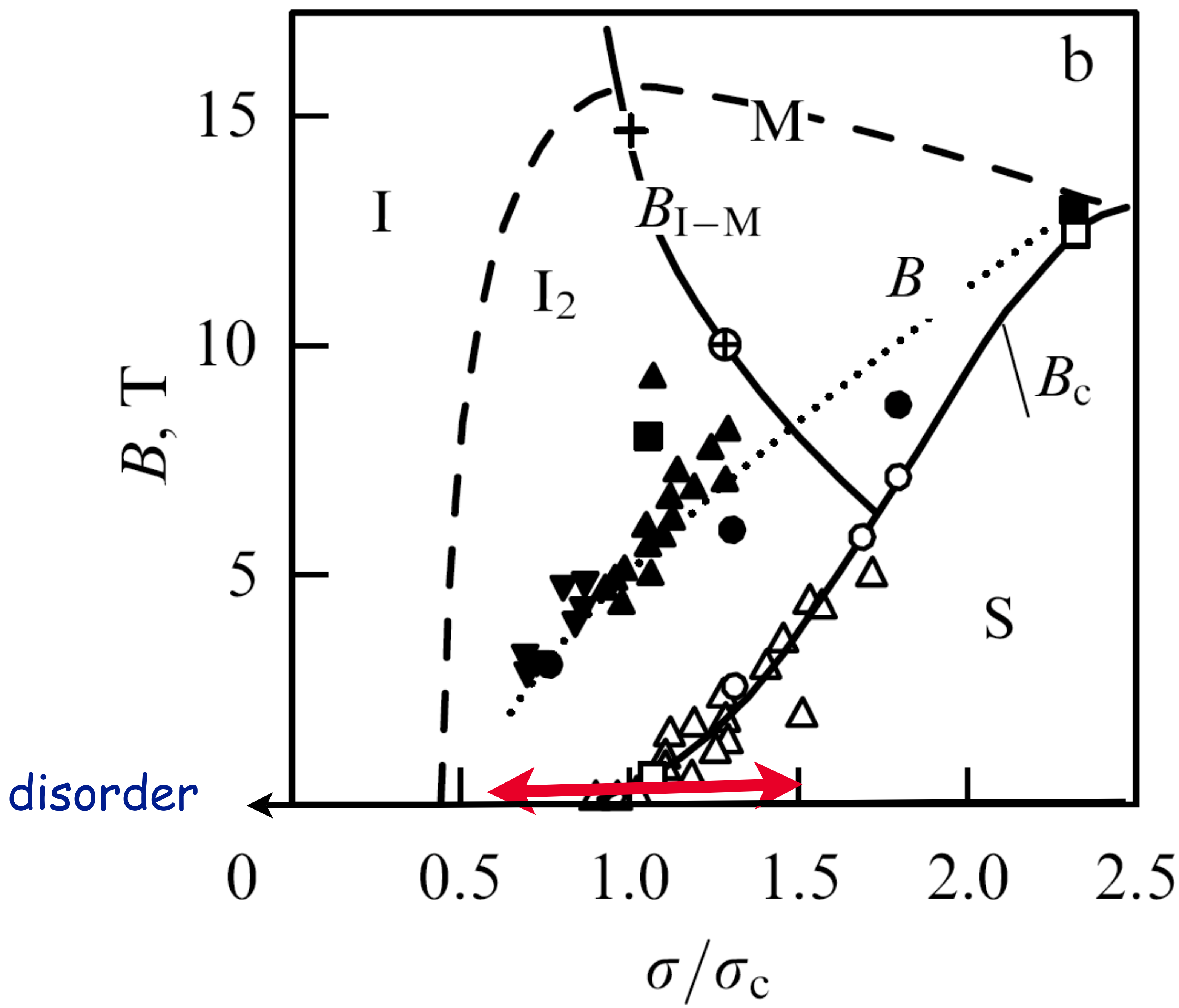
Simplification $\Psi(\mathbf{r}) \sim e^{-r^2/2R^2}$

$$\langle\Psi|\hat{H}|\Psi\rangle(R,\lambda) = E \quad \to \lambda(E,R)$$

$$\int d^3r \frac{U^2}{2\kappa^2} = \Phi(R,\lambda(E,R))$$

$$\boxed{\to min_R \to E = E(R) \qquad \to \nu(E) \sim e^{-\Phi(R)}}$$

Zero magnetic field



Phase diagram of a superconductor near the SIT transition.
The dashed line separates the region of existence of a glass state.
The dotted curve corresponds to a maximum of resistance.

V.F. Gantmakher and V. Dolgopolov, UFN (Russian Physics, Uspekhi, Jan. 2010.

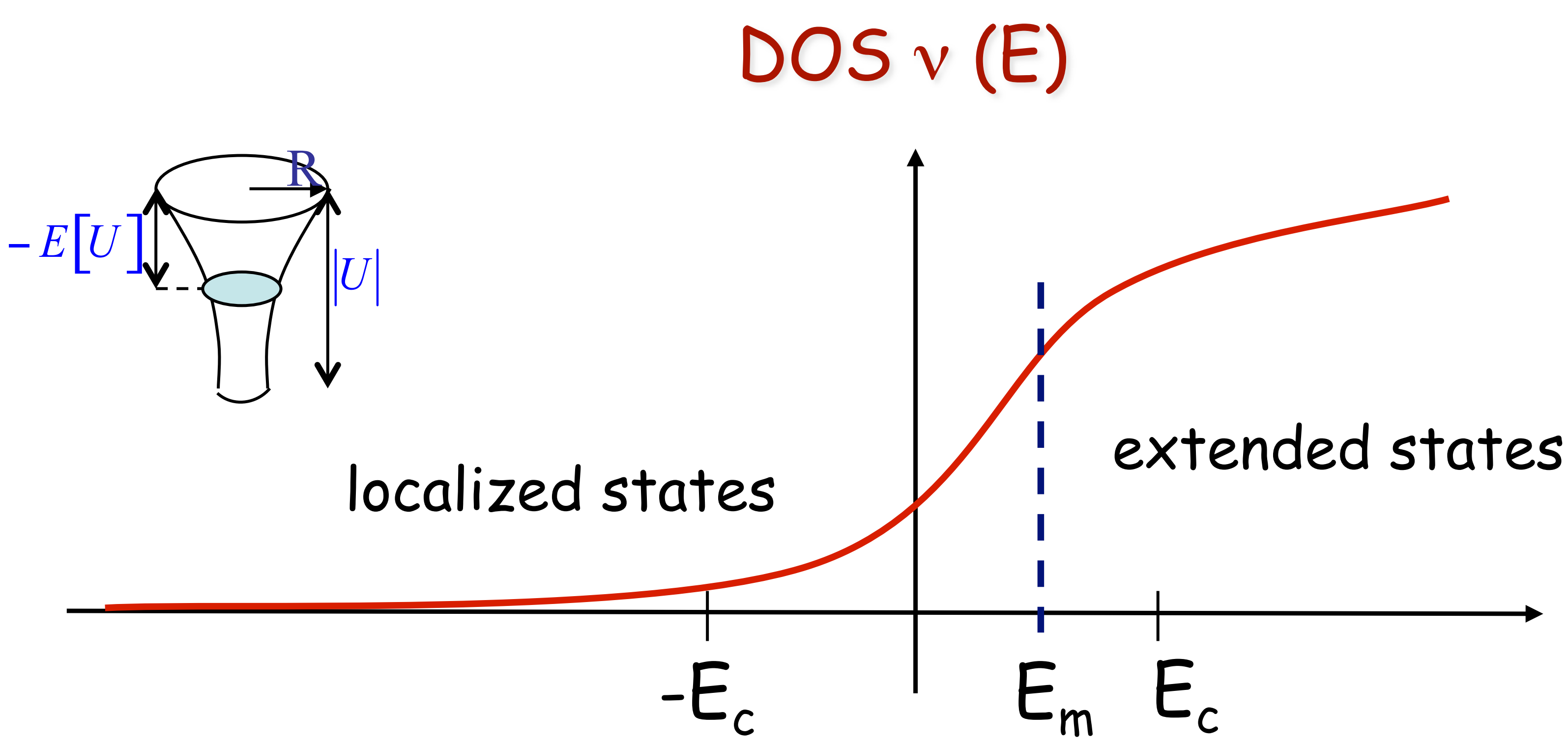
Simplification $\Psi(\mathbf{r}) \sim e^{-r^2/2R^2}$

$$\langle \Psi | \hat{H} | \Psi \rangle (R, \lambda) = E \quad \to \lambda(E, R)$$

$$\int d^3 r \frac{U^2}{2\kappa^2} = \Phi(R, \lambda(E, R))$$

$$\boxed{\to min_R \to E = E(R) \qquad \to \nu(E) \sim e^{-\Phi(R)}}$$

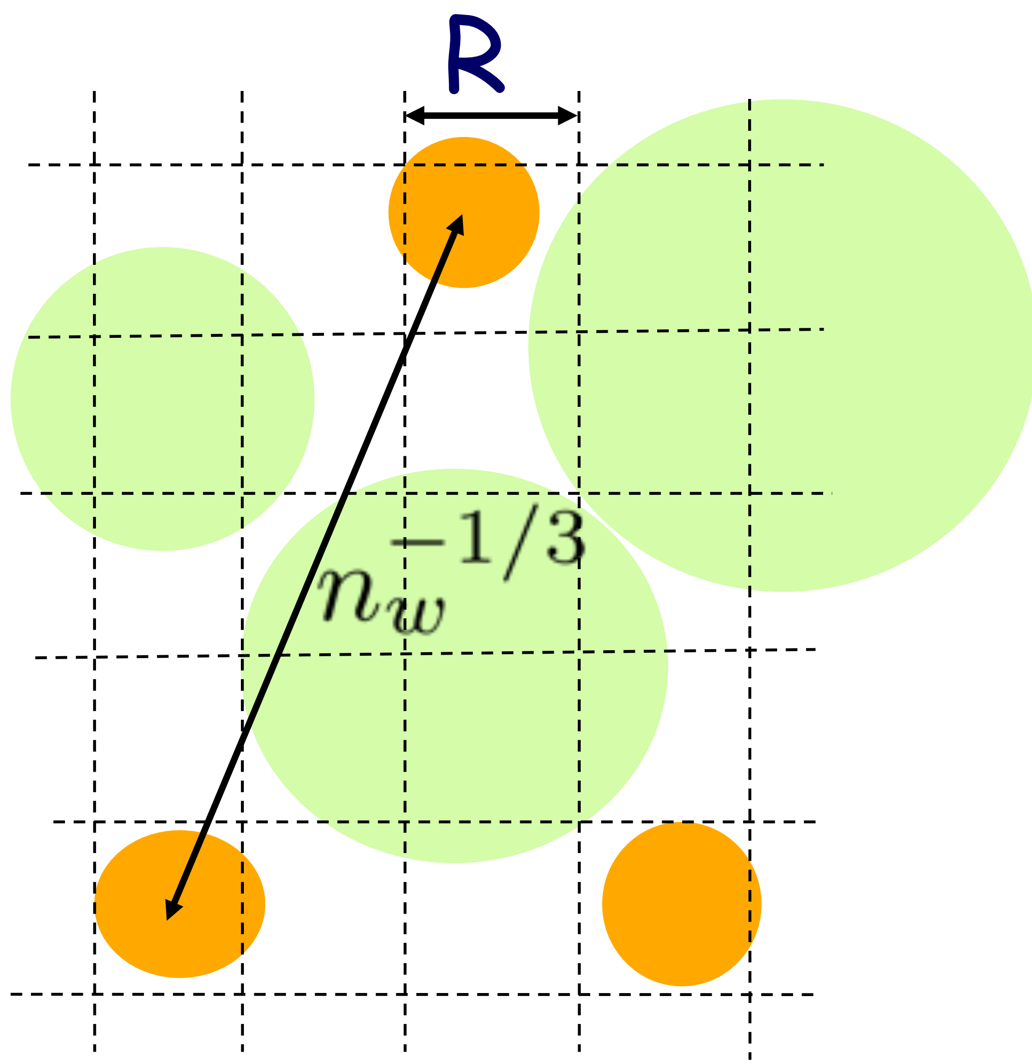
Zero magnetic field



$$\boxed{\begin{aligned} E(R) &= -\frac{\hbar^2}{2m_k R^2} \\ \nu(R) &\sim e^{-(\mathcal{L}_b/R)^{4-d}} \end{aligned}}$$

Zero magnetic field

Density of well with radius smaller than R $\ll \mathcal{L}_k$



$$\boxed{n_w(R) = \int_0^R dR\nu(R) \sim e^{-(\mathcal{L}_k/R)^{4-d}}}$$

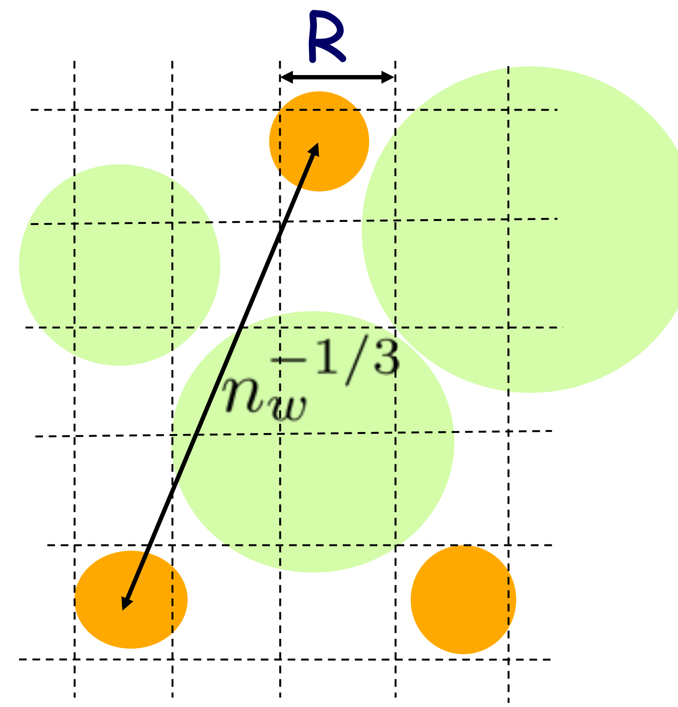
Tunneling amplitude t(R) between wells with radius < R :

$$t(R) = \exp\left(-\frac{1}{\hbar}\int |p|dr\right)$$

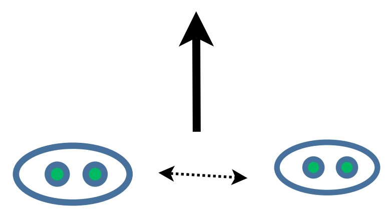
$$\frac{1}{\hbar}\int |p|dr \approx n_w^{-1/d}/R \sim e^{(\mathcal{L}_k/R)^{4-d}/d}$$

$$\boxed{t(R) \sim e^{-e^{(\mathcal{L}_k/R)^{4-d}/d}}}$$

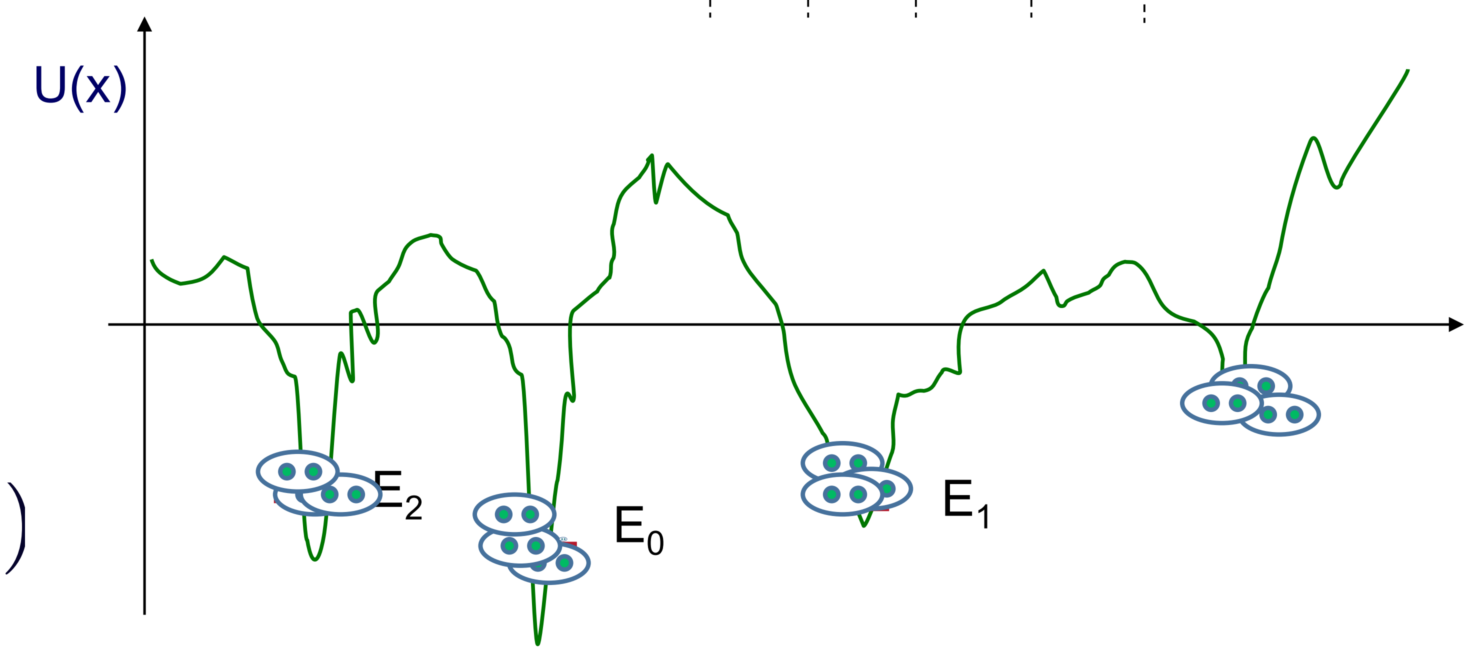
Fill now bosons into random potential



$$\mu_b(R) = -\frac{\hbar^2}{2m_k R^2} + g\frac{n_b}{R^d n_w(R)}$$



CP interaction



minimization

$$\Rightarrow R(n) \Rightarrow \mu(n)$$

$$\boxed{\begin{aligned} R(n) &\approx \mathcal{L}_b/(\ln(n_c/n_b))^{1/(4-d)} \\ \mu_b &= -\,\mathcal{E}_b \ln^{2/(4-d)}(n_c/n_b) \end{aligned}}$$

size of puddle

$$n_c = \frac{\hbar^2}{4m\mathcal{L}_b^2 g}$$

Babichenko[2]

Preliminary conclusions

$\Rightarrow$ At n << n_c the SC decays into fragments, $R(n) \approx \mathcal{L}_b/(\ln(n_c/n_b))^{1/(4-d)}$

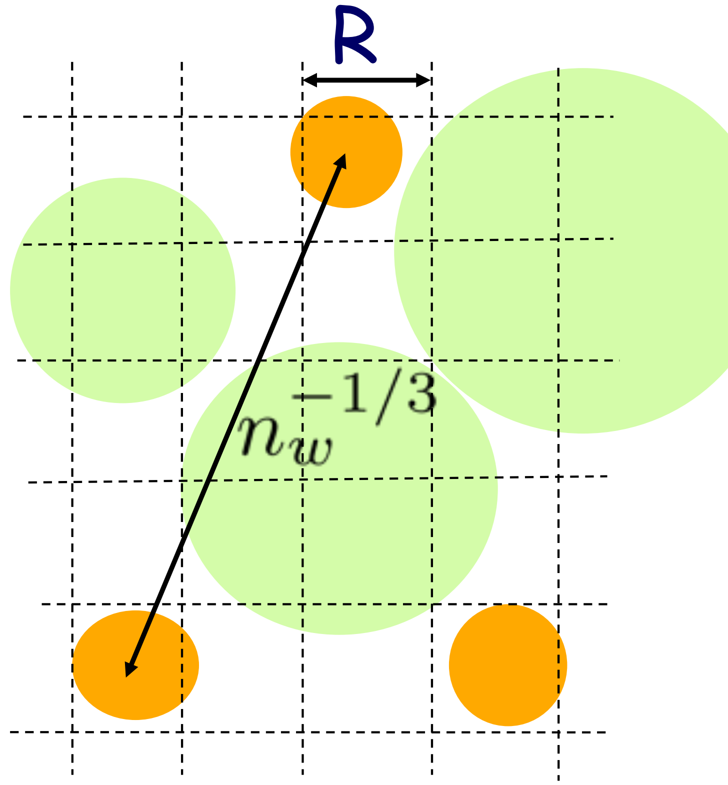
particle density in fragments $n_c = \dfrac{\hbar^2}{4m\mathcal{L}_b^2 g}$

$\Rightarrow$ tunneling exponentially suppressed $t(n) \sim e^{-c(n_c/n_b)^{1/d}}$

$\Rightarrow$ particle number in fragments well defined

$\Rightarrow$ phase uncertain, no phase coherence $\Rightarrow$ no SC

$\Rightarrow$ finite compressibility $\dfrac{\partial n}{\partial \mu} = \dfrac{n}{E_c}\ln(n_c/n)$ „Bose glass"

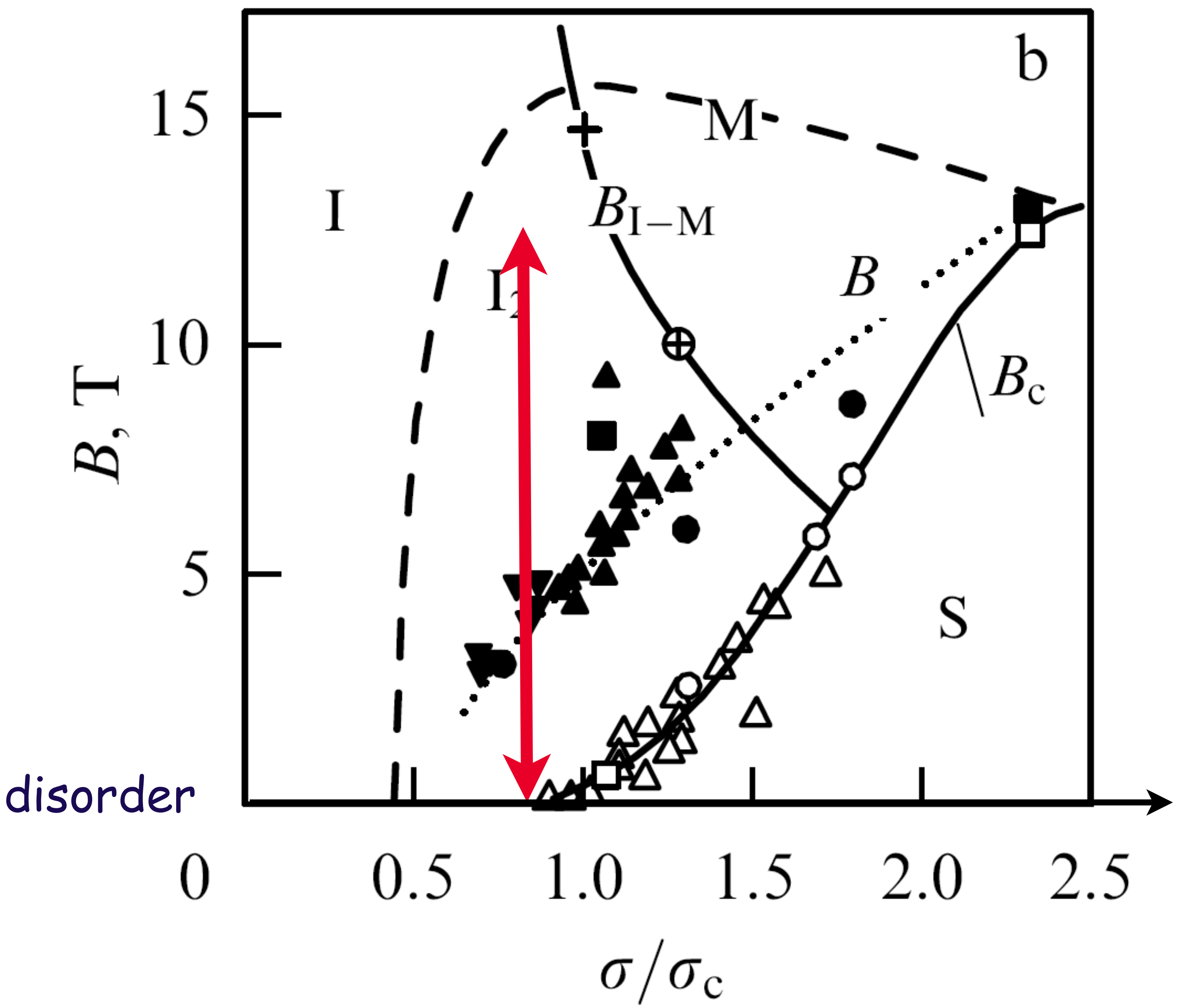


$\Rightarrow$ charged bosons VRH $\boxed{R(T) \sim e^{(T_0/T)^{1/(d+1)}} \qquad T_0 = \mathcal{E}_b \frac{n_c}{n_b}}$

For $n \approx n_c$ fragments merge $\rightarrow$ SIT

Model: random Josephson junction array

Extension to finite magnetic field



Phase diagram of a superconductor near the SIT transition.
The dashed line separates the region of existence of a glass state.
The dotted curve corresponds to a maximum of resistance.

Extension to finite magnetic field

Energy of wave function of radius R

$$E(R) = -\frac{\hbar^2}{2m_k R^2}\left(1 - \frac{3}{4}\frac{R^4}{\ell_k^4}\right) \qquad \psi_k(\mathbf{r}) = \pi^{-1/2} R_k^{-1} e^{-r^2/2R_k^2}$$

kinetic + potential energy   diamagnetic contribution

Density of states

$$\nu(R) \sim \mathcal{L}^{-d} \exp\left[-\left(\frac{\mathcal{L}_b}{R}\right)^{4-d}\left(1 - \frac{R^4}{4\ell_k^4}\right)^{(6-d)/2}\right]$$

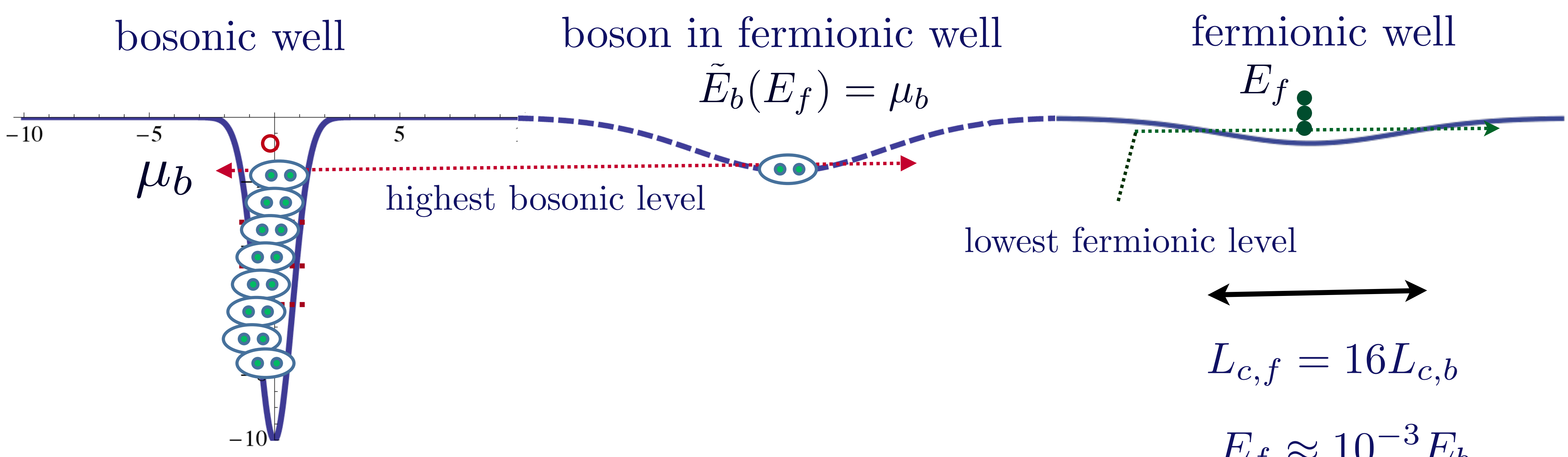
Lifshitz tail   Landau level

Increase magnetic field such that

$$\boxed{\mu_b - 2\Delta \geq 2\left(E_f + E_z\right)}$$

Application of magnetic field destroys some CPs: where to put the fermions?

Optimal fluctuation of random potential for bosons and fermions, respectively



Thin film in parallel field

$w \ll \ell_B = \sqrt{c\hbar/eB}$ $\longrightarrow$ Diamagnetic effect is negligible

SC to insulator transition happens at $n_b g \approx \mathcal{E}_b$ controlled by disorder

density of CP $n_b \approx \dfrac{m\Delta}{4\pi\hbar^2}$

energy of first fermion $E_f = (2/9)\mu_b$

$$\boxed{B^{\|}_{\mathrm{BFT}} \approx \frac{2\Delta - 5\mu_b/9}{g_e\mu_B} = B_c\Big[1 + \frac{5}{18}\kappa \ln(\kappa/\kappa_c)\Big].}$$

$B_c = 2\Delta/(g_e\mu_B)$ pair breaking field

$\kappa = \mathcal{E}_b/\Delta$ $\qquad \kappa_c = \dfrac{gm}{4\pi\hbar^2}$

How the density of fermions grows above the BFT?

Equilibrium condition: $\frac{d\varepsilon}{dn_f} = 0$ $\varepsilon\left(n, n_f\right)$- energy per unit area

$$d\varepsilon = \left(E_b - 2\Delta\right)dn_b + \left(E_f - \frac{g\mu_b B}{2}\right)dn_f$$

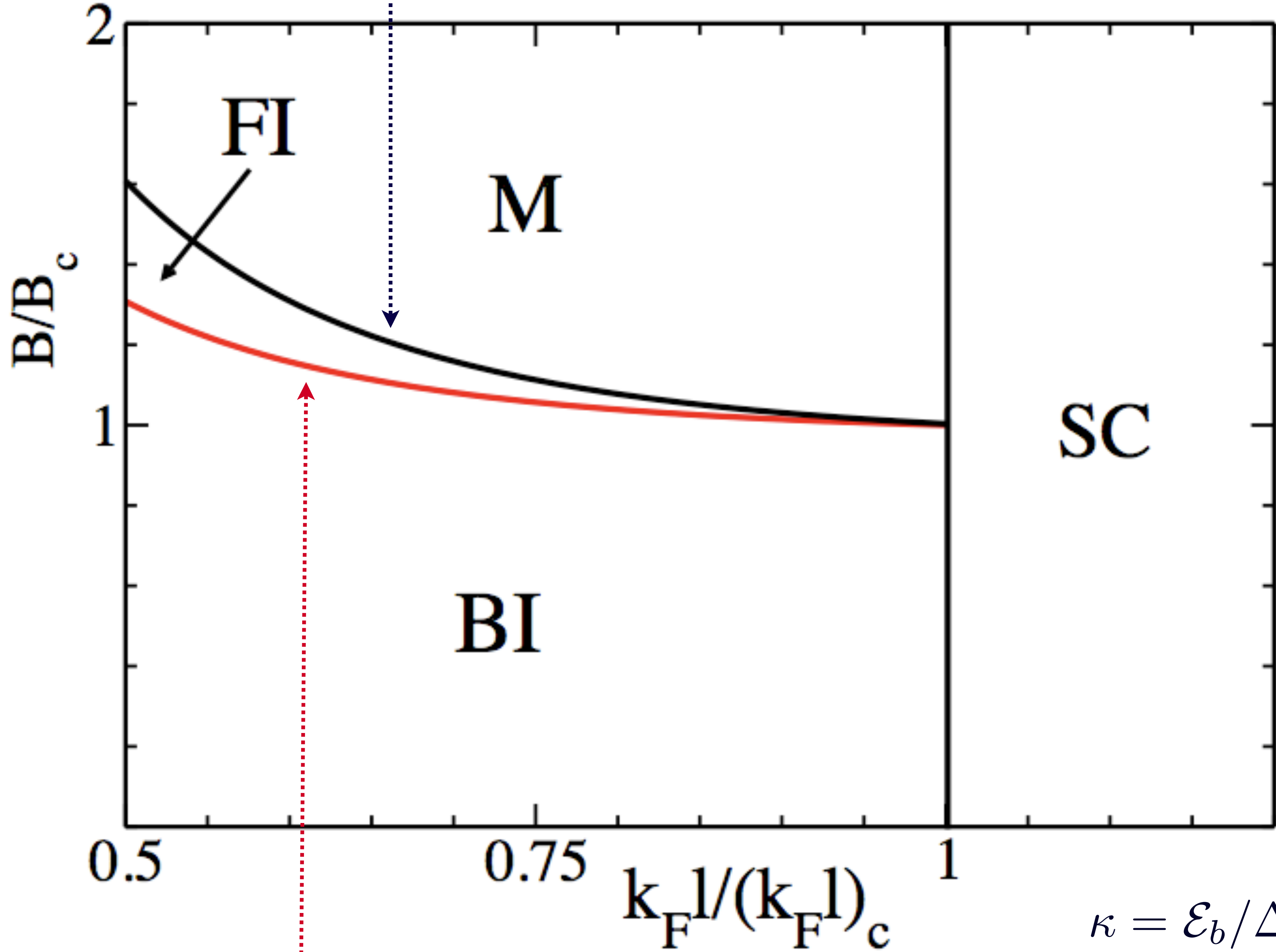
Conservation of number of electrons: $dn_b = -\frac{dn_f}{2}$

$\frac{d\varepsilon}{dn_f} = 0 \Rightarrow E_b - 2\Delta = 2\left(E_f - \frac{g\mu_B B}{2}\right)$ Now it is an equation determining n_f

Metal-Insulator Transition (MIT)

$n_f = n_{cf} = \mathcal{L}_f^{-2} \leq \frac{n_{cb}}{16}$ $n_b = n - \frac{n_{cf}}{2}$ $E_f \approx 0$

$$B^{\parallel}_{\rm MIT} \approx B_c \left[1 + \frac{\kappa}{2} \ln(\kappa/(\kappa_c - \kappa/32))\right].$$



$\kappa = \mathcal{E}_b/\Delta$

$\kappa_c = \frac{gm}{4\pi\hbar^2}$

$$B^{\parallel}_{\rm BFT} \approx \frac{2\Delta - 5\mu_b/9}{g_e\mu_B} = B_c\Big[1 + \frac{5}{18}\kappa \ln(\kappa/\kappa_c)\Big].$$

Thin film in perpendicular field

$\longrightarrow$ diamagnetic effects relevant

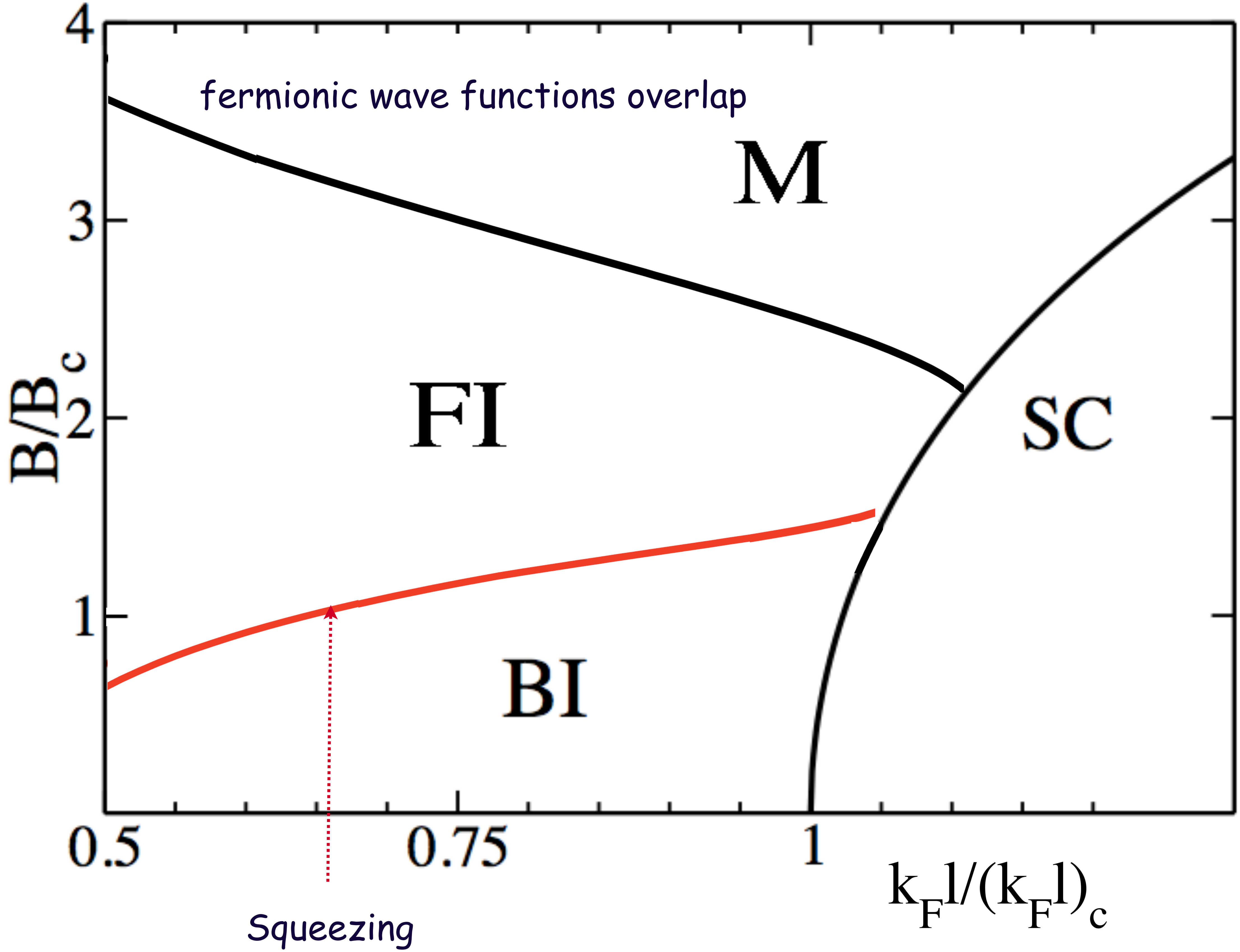
$$\gamma = 1 - m_0/mg_e > 0$$

Resistivity maximum $\quad B^{\perp}_{\mathrm{BFT}} \approx B_c[1 + \sqrt{(\gamma^{-1} - 1)\kappa/2\ln(\kappa/\kappa_c)}]/\gamma$

Wells more narrow than ξ , then suppression of Copper pairs: Squeezing
(alternatively: level spacing larger than gap i.e. at least on CP in well)

$$\boxed{B^{\perp}_{sq} \approx \frac{2B_{c2}}{c_1}\left[1 - (2c_1\kappa k_F l\ln(\kappa/\kappa_c))^{\frac{1}{2}}\right]^{1/2}}$$

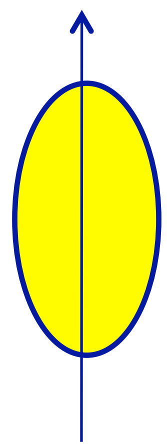
$$B_{c2} = B_c d/(8k_F l(1-\gamma)) = c_1\hbar c/(2e\xi^2), \quad c_1 \approx 0.69$$


fermionic wave functions overlap
M
FI
SC
BI
4
3
2
1
0.5
0.75
1
B/B_c
k_F l/(k_F l)_c
Squeezing

Three dimensional system

$$\psi_k\left(\mathbf{x}\right) = \pi^{-3/4} R_{k\perp}^{-1} R_{k\|}^{-1/2} \exp[-\rho/2R_{k\perp}{}^{2} + z^2/2R_{k\|}^{2}]$$

$$R_{k\|}^{-2} = R_{k\perp}^{-2} - R_{k\perp}^{2}/\ell_k^4$$

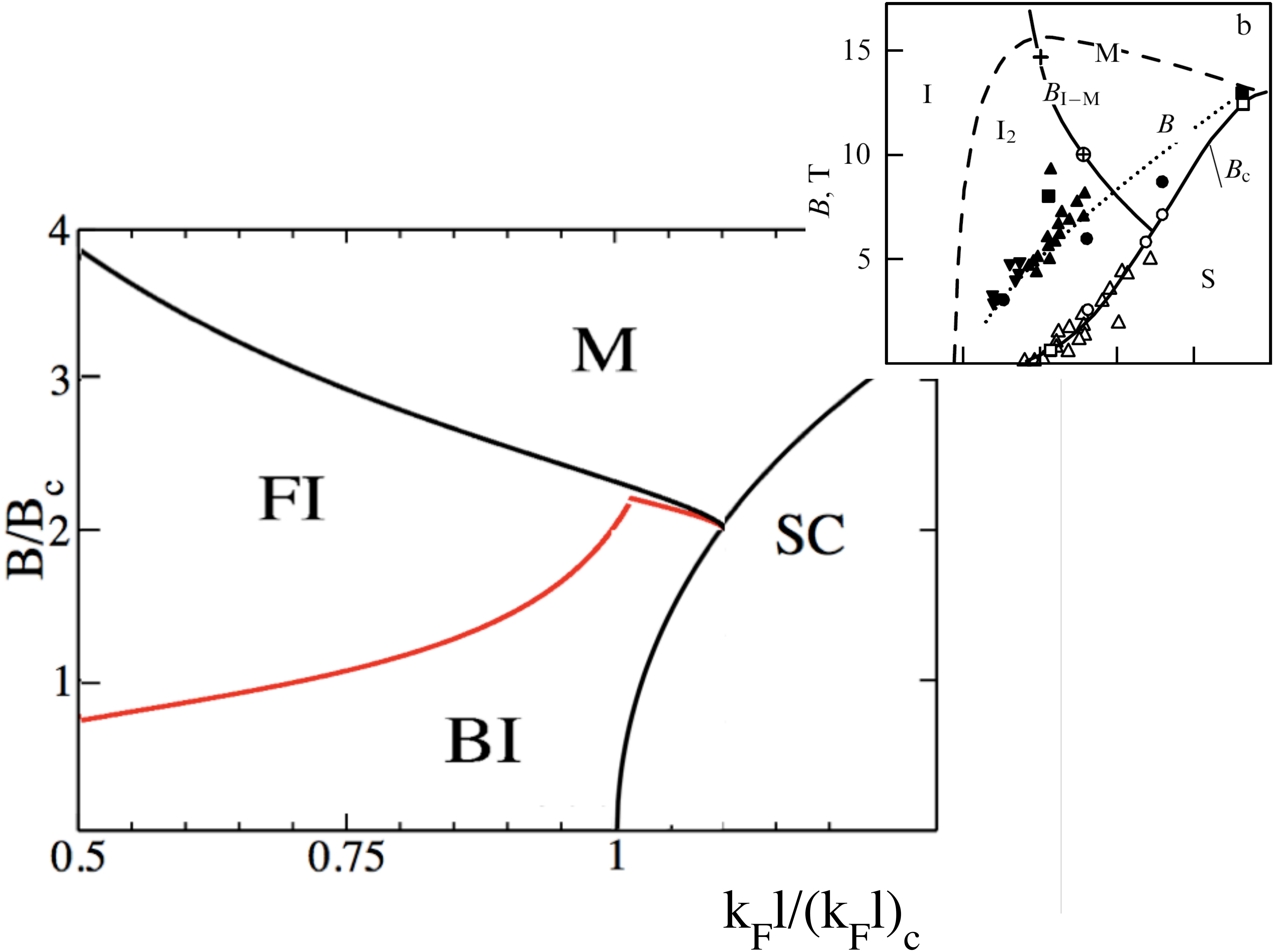


wave function strongly anisotropic

$$B_{\mathrm{MIT}}^{3D} \approx B_c\Big[1 + (\kappa/2)\ln^2(\kappa/\kappa_c)\Big]$$

$$B_{\mathrm{MIT}}^{3D} = \frac{B_c}{\gamma}\left[1 + \left(\frac{\kappa}{2}\ln^2\left(\kappa/\kappa_c\right)\right)^{1/3}\right]$$

$$\boxed{B_{\mathrm{sq}}^{3D} \approx \frac{B_{c2}}{\sqrt{\kappa k_F l}\,\ln(\kappa/\kappa_c)}\left[1 - \sqrt{\kappa k_F l}\,\ln(\kappa/\kappa_c)\right]^{1/2}}$$



4
3
2
1
B/B_c
0.5
0.75
1
$k_F l/(k_F l)_c$
M
FI
BI
SC
b
15
10
5
B, T
I
I_2
M
B_{I-M}
B
B_c
S

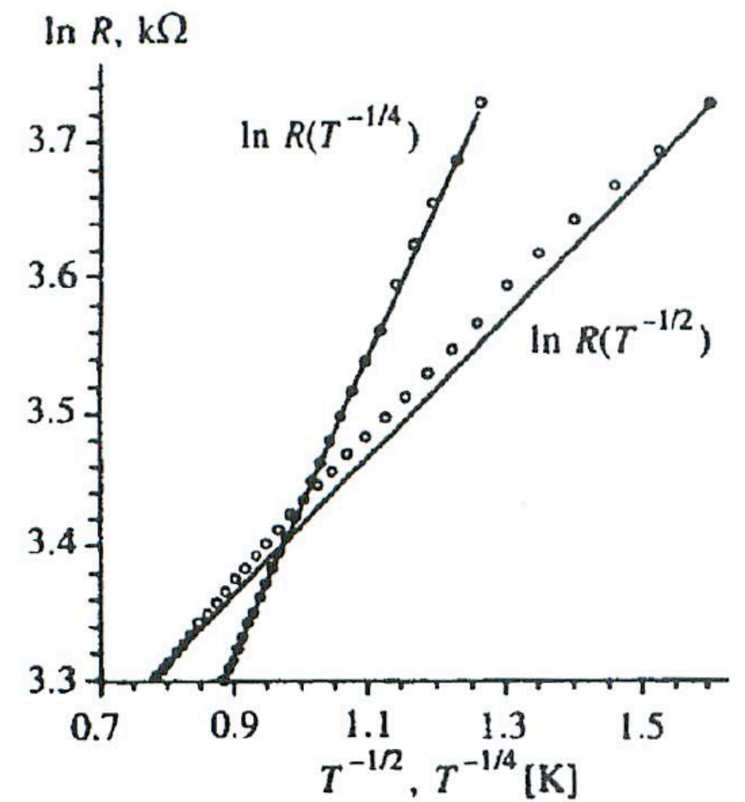
Resistivity

Assume Mott variable range hopping : $R = R_0 \exp[-(T_0/T)^{1/(d+1)}]$

Increase for increasing field since tail of wave function changes from simple exponential to Gaussian

Decrease for increasing field $B > B_{BFT}$ since T_0 for bosons is much larger than for fermions.

$$\frac{T_{0f}}{T_{0b}} \approx \frac{n_b}{n_f}\left(\frac{L_{cb}}{L_{cf}}\right)^{d+2} \sim \left(\frac{L_{cb}}{L_{cf}}\right)^2 = O(10^{-2})$$



Conclusions

- Phase diagram depends on dimensionality. In thin films it depends on the magnetic field direction.
- In all considered situations there are 4 interplaying phases: Bose Insulator, Fermi Insulator, Metal and Superconductor
- Transitions between them are due either to paramagnetic depairing or to squeezing of Cooper pairs by the random potential well in magnetic field
- Negative magnetoresistance appears due to proliferation of fermions which are weakly confined by the random field.
- In thick film or bulk the phase diagram does not depend on direction of magnetic field